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RESEARCH ON THE INVERSE PROBLEM OF SCATTERING

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ITEM #20, CONTINUED: areas for which the Gelfand-Levitan equation was valid. Thus we (a) set up generalizations of the Gelfand-Levitan equation for the one-dimensional and radial Schroedinger equations, (b) provided a variational principle, and (c) extended inverse techniques for non-Hermitian operators and/or non-local potentials. We were careful to provide explicit examples in order to assure ourselves that the procedures were not empty.

As we became more adept at solving inverse problems explicitly for certain kinds of spectral data, we were able to provide interesting examples and counter-examples for conjectured theorems. Thus we were able to show that scattering data generally does not tell one whether a scattering potential is local or, if it is local, gives it uniquely without prior knowledge of the range of the potential.

As is well-known, there is a dearth of scattering potentials, even for the one-dimensional and radial Schroedinger equations, for which the Schroedinger equation can be solved in terms of elementary functions. We have provided many new solvable potentials. In addition to providing potentials for scattering theory, we have shown how potentials with a purely or partially discrete spectrum can be changed to give solvable potentials for the same or altered discrete spectrum. In particular, we have obtained potentials for anharmonic oscillators whose solutions involve elementary functions only.

From the beginning we have regarded applications as one of our objectives. It has been possible to show that linear filter theory, used to remove noise in a variety of situations, involves an equation which is very close to the Gelfand-Levitan equation. In one of our published works done under the grant, we have stressed the analogy. It now appears that the Gelfand-Levitan equation for the one-dimensional and radial Schroedinger equations are identical to filter equations for important classes of applications. We hope to pursue this matter in the extension of the grant.

Another area of application is the design of acoustic media and electric transmission lines with prescribed transmission properties to carry signals. We have recently given several indices of refraction for which the acoustic and transmission line equations can be solved in terms of elementary functions. We have thereby added considerably to the small number of indices for which the equations can be solved exactly and in closed form.

Some of our most recent work on altering the spectrum of a particle in a box will certainly have some applications in one-dimensional systems. Certain polymers and crystals can be considered to be one-dimensional systems. We expect that some of our potentials will be useful in understanding and perhaps anticipating some properties of such systems.

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## 1.0 INTRODUCTION AND SUMMARY

The present section constitutes a brief discussion of the aims and achievements of the Principal Investigator during the term of the grant. One may consult interim reports for details.

As one might expect, as the research progressed, the aims of the research changed. Initially our principal concern was to obtain a better understanding of the fundamental ideas of inverse scattering and spectral theory, particularly in one dimension where the principal mechanism for obtaining potentials from spectral data, namely the Gelfand-Levitan, was well established. Toward this end we generalized some of the earlier results and added new areas for which the Gelfand-Levitan equation was valid. Thus we (a) set up generalizations of the Gelfand-Levitan equation for the one-dimensional and radial Schroedinger equations, (b) provided a variational principle, and (c) extended inverse techniques for non-Hermitian operators and/or non-local potentials. We were careful to provide explicit examples in order to assure ourselves that the procedures were not empty.

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Some of our most recent work on altering the spectrum of a particle in a box will certainly have some applications in one-dimensional systems. Certain polymers and crystals can be considered to be one-dimensional systems. We expect that some of our potentials will be useful in understanding and perhaps anticipating some properties of such systems.

A subject of our concern which is more properly related to the direct scattering problem, and which nonetheless has direct Air Force applications, is the calculation of the resonance scattering cross-section of atomic hydrogen for the Lyman-alpha radiation. These cross-sections are important

in the calculation of spectral line formation in the upper atmosphere of some atomic oxygen lines which lie close to the hydrogen line. The results also should prove useful in determining the range of atomic guns which use hydrogen atoms or hydrogenic atoms (such as lithium) as projectiles and detecting possible enemy usage of hydrogen atoms as a weapon. Surprisingly, our calculation of the atomic resonance cross-section is the first for any atom, though the general theory has been available since the early days of quantum mechanics. However, we shall not pursue this matter further under the grant renewal unless we are requested to do so.

## 2.0 INTERACTIONS WITH OTHER SCIENTISTS

During the period of the grant we have had numerous discussions with mathematicians and physicists about problems arising in inverse scattering. Among those with whom we have had particularly close contact and with whom we have collaborated with are Dr. P. B. Abraham of the Naval Underwater Systems Center, Prof. B. DeFacio of the University of Missouri, Prof. S. K. Mitter of the Massachusetts Institute of Technology, Prof. J. M. Cohen of the University of Pennsylvania and Prof. R. T. Prosser of Dartmouth College. In addition there have been more casual contacts with others such as Prof. H. Haus of MIT, Prof. P. Caines of Harvard, and Prof. E. Trubowitz then of MIT. We also have attended and in some cases helped organize seminars at MIT, Los Alamos, Naval Research Laboratory, University of Arizona and elsewhere.

We have given invited talks at New York University, Tufts University, Boston College, Air Force Geophysics Laboratory, Iowa State University, Princeton University, University of Pennsylvania, Naval Research Laboratory, Naval Underwater Systems Center, Clark University, University of Connecticut.



### 3.0 PUBLICATIONS

1. A Generalization of the Gelfand-Levitan Equation for the One-Dimensional Schroedinger Equation, J. Math. Phys., 18, 2243 (1977).
2. A Variational Principle for the Gelfand-Levitan Equation and the Korteweg-deVries Equation (with M. Kanal), J. Math. Phys., 18, 2445 (1977).
3. A Generalization of the Direct and Inverse Problem for the Radial Schroedinger Equation, Studies in App. Math., 58, 187 (1978).
4. A Generalization of the Inverse Scattering Problem for the Korteweg-deVries Equation. A Variational Principle, in Solitons in Action (K. Lonngren and A. Scott, Editors), Academic Press (1978).
5. An Example of the Effect of Rescaling of the Reflection Coefficient on the Scattering Potential for the One-Dimensional Schroedinger Equation, Studies in App. Math., 60, 177 (1979).
6. Gelfand-Levitan Equations with Comparison Measures and Comparison Potentials, J. Math. Phys., 20, 2047 (1979).
7. A Variational Principle for the Linear Filter Matrix and an Interpretation for the Maximum Value of the Functional (with M. Kanal and S. K. Mitter).
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10. Changes in Potentials due to Changes in the Point Spectrum. Anharmonic Oscillators with Exact Solutions (with P. B. Abraham), Phys. Rev. A, 22, 1333 (1980).
11. Resonance Scattering of Lyman-Alpha Radiation by Hydrogen in the Ground State, Phys. Rev. A, 22, 2069 (1980).
12. Two Distinct Potentials with No Bound States Can Have the Same Scattering Operator: A Nonuniqueness in Inverse Spectral Transformations (with P. B. Abraham and B. DeFacio), Phys. Rev. Letters, 46, 1657 (1981).